Example 1

Consider the random variable X, a person's height, drawn from a population where the X is distributed normally with mean of 66 and variance of 20.

What is P(X>66.5)?

$X \sim N(66, 20)$

$$P(X > 66.5) = P\left(\frac{X-\mu}{\sigma} > \frac{66.5-60}{\sqrt{20}}\right)$$
$$= P(Z > 0.5/\sqrt{20})$$
$$= 1 - P(Z < 0.1118)$$

z	0	0.01	0.02	0.03	0.04	0.05
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368

$1 - 0.5438 = 0.4562$

Suppose we randomly select 125 individuals from that population. What is the probability that the mean height is at least 66.5 inches? Does it matter whether X is distributed normally?

$$CLT \Rightarrow \bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$$

$$P(\bar{X} > 66.5) = P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} > \frac{66.5-66}{\sqrt{20/125}}\right)$$
$$= P\left(Z > \frac{0.5}{0.4}\right)$$
$$= P(Z > 1.25) = 1 - \Phi(1.25)$$

0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406

$1 - 0.8944$

0.1056

Example 2

For a class project, a student wants to check whether final exam stress raises blood pressure of the students in her class. When they are not under any stress, college students have an average systolic blood pressure of 120mm Hg. For the 50 students in her class, the average blood pressure on the day of the final exam is 125.2 mm Hg with a standard deviation of 12mm Hg.

What is the p-value of her test?

1. $H_0: \mu_0 \le 120$

   $H_1: \mu_0 > 120$

2. $t = \frac{\bar{x}-\mu_0}{s/\sqrt{n}} = \frac{125.2-120}{12/\sqrt{50}} = \frac{5.2}{1.697} = 3.06$

(optional)

notice the same as t

reject at standard significance levels

3. $P(\bar{x} > 125.2 \mid H_0 \text{ true}) = P\left(\frac{\bar{x}-\mu}{s/\sqrt{n}} > \frac{125.2-120}{12/\sqrt{50}}\right) = P(Z > 3.06)$

2.8	0.9974	[illegible]	0.9976	0.9977	0.9977	0.9978
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989

$1 - 0.9989 = 0.0011$

probability more extreme than what you found.

What should the student conclude about whether final exam stress ~~raises~~ associated w/ higher blood pressure?

yes, blood pressure is higher and statistically significant different. at traditional levels of significance

Example 3

Another student wants to check whether final exams raise students' blood pressure more for chemistry or economics classes. For the 50 students in her economics class, the average blood pressure on the day of the final exam is 125.2 mm Hg with a standard deviation of 12mm Hg. For the 100 students in her chemistry class, the average blood pressure is 120 mm Hg with a standard deviation of 24mm Hg.

Conduct the hypothesis test for whether final exams raise students' blood pressure more for chemistry or economics students. Set up a null and alternative hypothesis, report your t-statistic, and your conclusion. Set $\alpha = 0.10$, use normal dist as

1. $H_0: \mu_e = \mu_c \Rightarrow H_0: \mu_e - \mu_c = 0$

$H_1: \mu_e \neq \mu_c \Rightarrow H_1: \mu_e - \mu_c \neq 0$

3. $$t = \frac{\bar{x}_e - \bar{x}_c - d_0}{SE[\bar{x}_e - \bar{x}_c]} = \frac{125.2 - 120}{2.93} = 1.77$$

4. $C_{\alpha/2} = C_{0.05} = 1.645$

2. $$SE[\bar{x}_e - \bar{x}_c] = \sqrt{\frac{s_e^2}{n_e} + \frac{s_c^2}{n_c}} = \sqrt{\frac{12^2}{50} + \frac{24^2}{100}} = \sqrt{2.88 + 5.76} = 2.93$$

1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.

5. reject at 10% level

[Side note] Does this mean that economics exams are more stressful than chemistry? Why or why not?

not necessarily
1. different underlying populations
2. different exam conditions