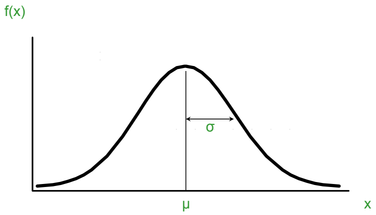
Review: Normal probability distributions

EC200: Econometrics and Applications

Normal distribution

- Location determined by the mean, μ.
- Spread determined by standard deviation, σ.
- Bell-shaped & symmetrical
- Mean = median = mode
- Infinite range, $-\infty < x < \infty$



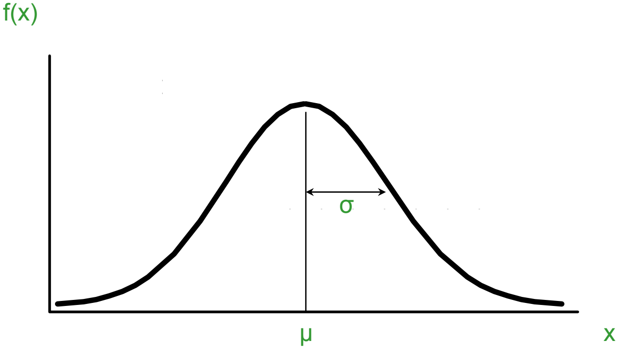
Normal distribution

- Distribution of sample means approach normal distribution with "large" sample size (Central Limit Theorem)
- Easy to compute probabilities!

A family of distributions

- Each distribution characterized entirely by μ and σ.
- We write the following for each distribution:

$$X \sim N(\mu, \sigma^2)$$



Normal PDF

Normal probability density function:

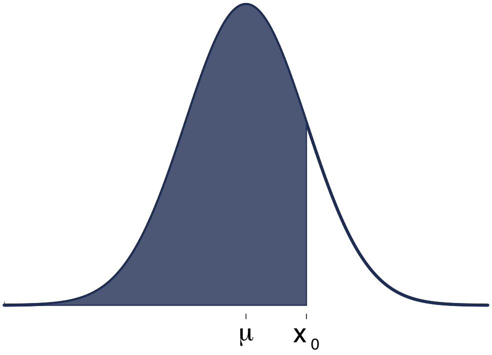
$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/2\sigma^2}$$

This is difficult to work with directly! We will use probability tables.

Normal CDF

For $X \sim N(\mu, \sigma^2)$, the cumulative distribution function is:

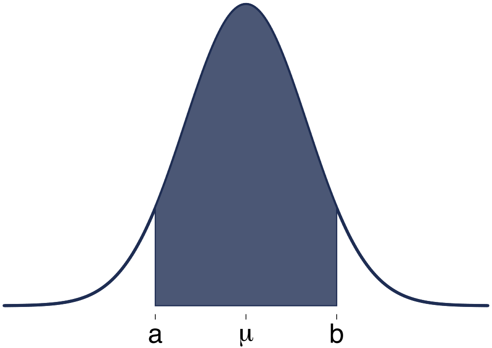
$$F(x_0) = P(X \leq x_0)$$



Finding normal probabilities

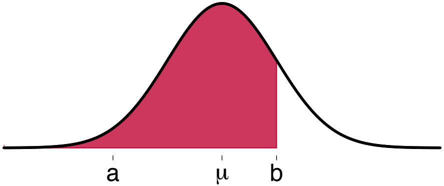
The probability for a range of values is measured by the area under the curve:

$$P(a < X < b) = F(b) - F(a)$$

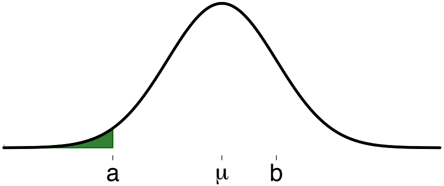


Finding normal probabilities

$$F(b) = P(X < b)$$



$$F(a) = P(X < a)$$



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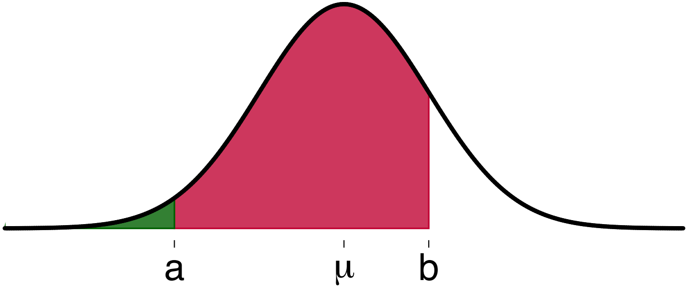
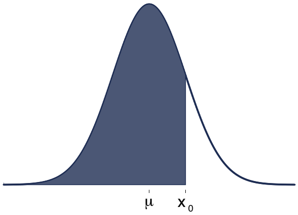


Finding normal probabilities

Things to note:

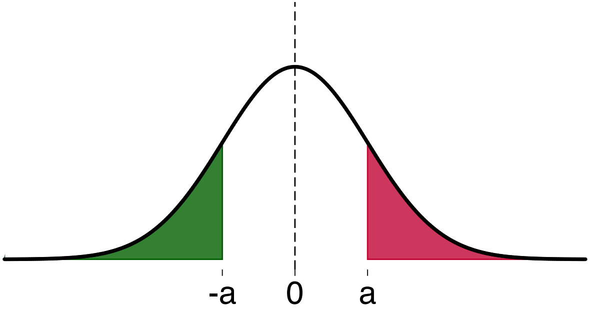
- $P(X \leq x_0) = P(X < x_0)$
- $P(X < x_0) = 1 - P(X > x_0) \Rightarrow$
  $P(X > x_0) = 1 - P(X < x_0)$



Finding normal probabilities

Things to note:

- $P(X < -a) = P(X > a)$.



Recap: Linear functions of random variables

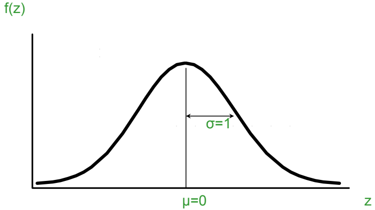
Special case: standardized random variable.

$$Z = \frac{X - \mu_X}{\sigma_X}$$

which has $\mu_Z = 0$ and $\sigma_Z^2 = 1$

The standard normal distribution

- *Any* normal distribution can be transformed into the standardized normal distribution ($Z \sim N(0,1)$):



- We transform X units into Z units by subtracting the mean of X and dividing by its standard deviation:

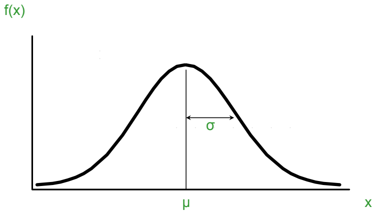
$$Z = \frac{X - \mu_X}{\sigma_X}$$

Example: normal probabilities

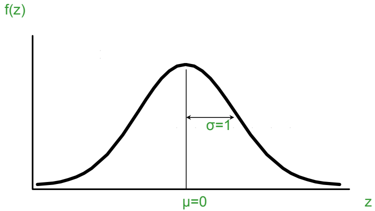
Example 1

If $X \sim (100, 50^2)$, what is the Z-value for $X = 200$?

Comparing X and Z units

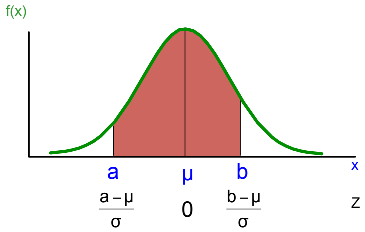


$\Leftrightarrow$



Note that the distribution is the same, only the scale has changed.
We can express the problem in original units (X) or standardized units (Z)

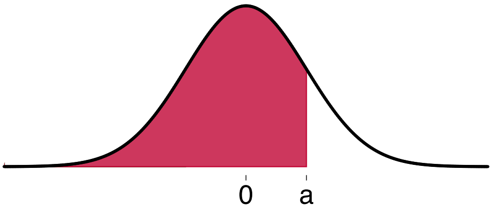
Finding normal probabilities



$$P(a < X < b) = P(\frac{a-\mu}{\sigma} < Z < \frac{b-\mu}{\sigma})$$
$$= F(\frac{b-\mu}{\sigma}) - F(\frac{a-\mu}{\sigma})$$

Standard normal distribution table

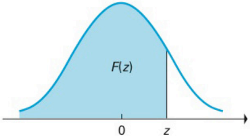
- The standard normal distribution table (available on Blackboard) shows values of the cumulative normal distribution function.
- For a given Z-value a, the table shows $F(a)$



Standard normal distribution table

APPENDIX TABLES

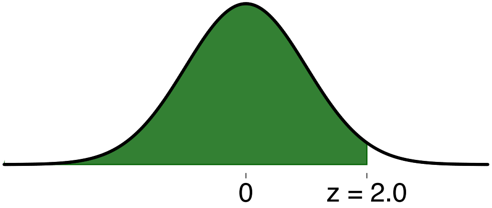
Table 1 Cumulative Distribution Function, $F(z)$, of the Standard Normal Distribution Table



z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549

Finding normal probabilities

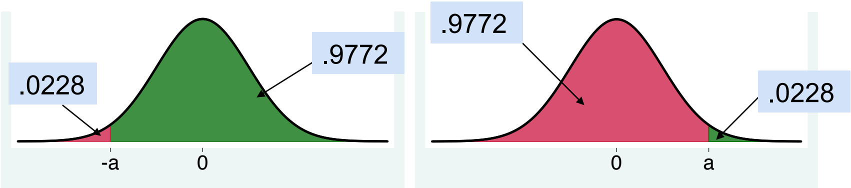
$$P(Z < 2.00) = 0.9772$$



Finding normal probabilities

For negative Z-values, recall that the distribution is symmetric:

$$P(Z < -a) = 1 - P(Z < a)$$



Type A: find probabilities, given $X \sim N(a, b)$

Example: A cupcake factory's daily production of cupcakes is normally distributed, with an average of 5,100 cupcakes per day and a standard deviation of 1,200 cupcakes. What is the probability that the factory produces more than 6,000 cupcakes tomorrow?

1. Draw normal curve for the problem in terms of X
2. Translate X-values to Z-values
3. Break into pieces of the form $F(Z < z)$
4. Use the cumulative normal table

Type B: find X-value, given probabilities

Example: A cupcake factory's daily production of cupcakes is normally distributed, with an average of 5,100 cupcakes per day and a standard deviation of 1,200 cupcakes. There is a 10% chance that the factory produces fewer than how many cupcakes tomorrow?

1. Find the Z-value for the known probability
2. Convert to X units using the formula:

$$X = \mu + Z\sigma$$

General rounding guidelines

Common z-values:

$F(z)$	0.90	0.95	0.975	0.99
z	1.282	1.645	1.960	2.326